

Names: \_\_\_\_\_

Score: \_\_\_\_\_/140

# **Aimer's Astronomy Answer Key**

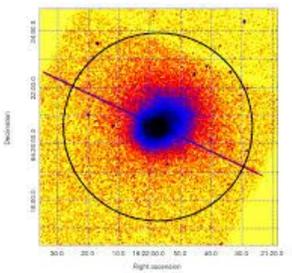
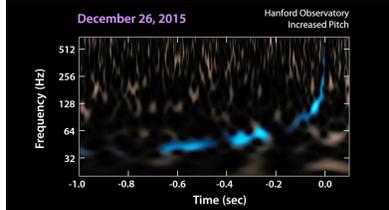
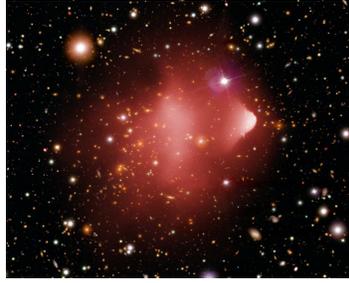
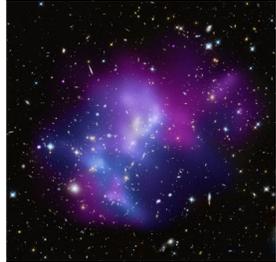
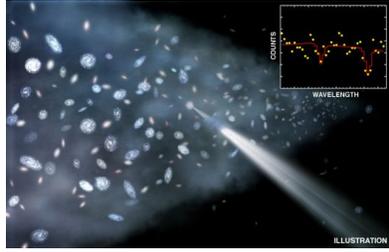
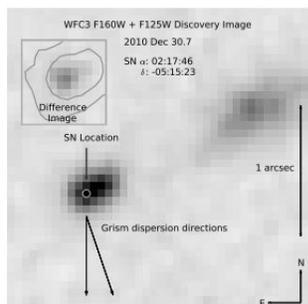
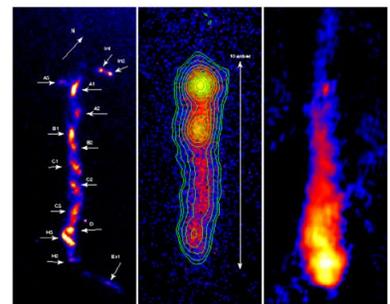
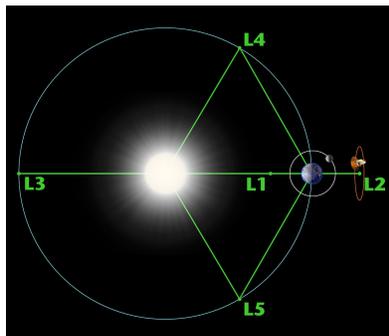
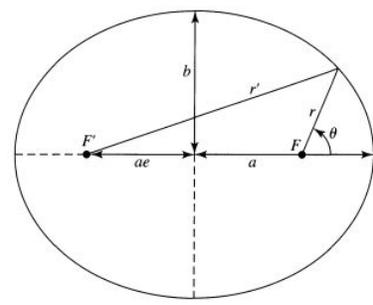
## **Division C**

### **SSSS 2020**

#### **Notes:**

- This test is based on the Division C 2020 rules for Astronomy.
- You may use the resources detailed in section 2 of the Division C 2020 rules.
- There is no JS9 on this test, so please do not access the internet during this test.
- Use at least 3 significant figures for answers in the Calculations section.
- Please do not use this test for tournaments or tryouts.
- If you have feedback or questions, please contact me on Discord (Aimer#8317) or on the forums (Aimer).

Image Sheet:

<p>A:</p> 	<p>B:</p> 	<p>C:</p> 
<p>D:</p> 	<p>E:</p> 	<p>F:</p> 
<p>G:</p> 	<p>H:</p> 	<p>I:</p> 
<p>J:</p> 	<p>K:</p> 	<p>Empty</p>

## Section 1: DSOs (42 pts)

1.
  - a. Identify the DSO in image A and the wavelength the image is in. (2 pts) **H1821+643 (+1), X-ray (+1)**
  - b. What is the mass of this DSO, in solar masses? (1 pt) **30 billion solar masses**
  - c. True / **False** (circle one): This DSO is currently the most massive black hole with a precisely measured mass. (1 pt) **\*\*\*In 2014, it WAS the most massive black hole with a precisely measured mass, but more massive black holes have been since measured with great precision**
  - d. How is this DSO used to detect the Warm-Hot Intergalactic Medium? (2 pts) **Astronomers are looking at the X-ray spectrum emitted by this quasar (+1), and looking for dips in the spectrum that would suggest certain wavelengths are being absorbed by the WHIM's hot gas components (+1).**
2.
  - a. Identify the DSO in image B. (1 pt) **GW151226**
  - b. What does this DSO suggest about binary black holes throughout the universe? (1 pt) **It suggests that there is a large population of binary black holes in the universe that will produce frequent mergers.**
  - c. What percentage of the final mass was radiated away as energy? Is this consistent with the percentage of mass radiated away from the first detected black hole merger (GW150914)? (2 pts) **Roughly 4.6% of the mass was radiated away (+1). This is consistent with the findings from GW150914 (+1).**
3.
  - a. Identify the DSO in image C and the wavelengths the image is in. (2 pts) **Bullet Cluster (1E 0657-558) (+1), X-ray and optical (+1)**
  - b. Briefly explain why this DSO provides the best evidence for dark matter. (2 pts) **Gravitational lensing studies show that the lensing effect is strongest in the two separated regions near the visible galaxies, however the majority of the baryonic matter**

## Answer Key

is not located there (+1). This implies that there is a significant amount of dark matter near the edges that is contributing to the lensing effect (+1).

4.
  - a. Identify the DSO in image D and the wavelengths the image is in. (2 pts) **MACS J0717.5+3745 (+1), X-ray and optical (+1)**
  - b. Which subcluster is moving quickly compared to the others? (1 pt) **Subcluster B**
  - c. The subcluster from question 4b exhibits an effect that distorts the Cosmic Microwave Background radiation. What is the name of this effect? (1 pt) **Sunyaev-Zel'dovich effect**
  - d. This DSO is the largest known \_\_\_\_\_ **gravitational lens** \_\_\_\_\_. (1 pt)
5.
  - a. Identify the DSO in image E. (1 pt) **H2356-309**
  - b. What is the distance to this DSO, in light years? (1 pt) **2 billion ly**
  - c. Astronomers are using this DSO to detect the Warm-Hot Intergalactic Medium by looking for dips in the DSO's spectrum caused by the WHIM absorbing certain wavelengths. Which element inside the WHIM is responsible for these dips? (1 pt) **Oxygen**
6.
  - a. Identify the DSOs in image F, and identify the wavelength of the smaller insets on the right. (2 pts) **PSS 0133+0400 and PSS 0955+5940 (+1), X-ray (+1)**
  - b. What do these DSOs suggest about dark energy over time? (1 pt) **Dark energy may not have been a constant throughout the history of the universe, as it may have changed in the past.**
  - c. A new method involving both UV and X-ray light was used to measure the distance to these DSOs. In order for this method to work, what must happen to some of the UV light? (2 pts) **Some of the UV light must be boosted up to X-ray wavelengths (+1) by colliding with electrons in a cloud of hot gas. (+1)**
- 7.

## Answer Key

- a. Identify the DSO in image G and the wavelengths the image is in. (2 pts) **M87 (+1), X-ray and radio (+1)**
  - b. What does this DSO's relativistic jet suggest about active galaxies? (1 pt) **It suggests that all active galaxies are the same, just viewed from different perspectives.**
  - c. This DSO is notable for its large number of globular clusters. How might this DSO have accumulated some of these clusters? (1 pt) **M87 is located near the center of the Virgo cluster, so it may have taken globular clusters from nearby galaxies with its gravity.**
  - d. What is the name of the technique used to image the supermassive black hole at the center of this DSO? (1 pt) **Very Long Baseline Interferometry (VLBI)**
- 8.
- a. Identify the DSO in image H. (1 pt) **SN UDS10Wil**
  - b. What is the distance to this DSO? What was the approximate scale factor of the universe ( $a$ ) when this DSO's light was emitted? (2 pts) **10.5 billion light years (+1),  $a = 0.33$  (+1)**
  - c. What does this DSO suggest about the two different models of Type Ia Supernovae? (1 pt) **The decline in the rate of type Ia supernova blasts between 7.5 billion years ago and 10 billion years ago suggests that most type Ia supernovae are triggered by the merger of white dwarfs, which could have implications for the cosmological distance ladder.**
- 9.
- a. Identify the DSO in image I and list the wavelengths of each section of the image, from left to right (hint: the image shows a specific part of the DSO, not the entire object). (3 pts) **3C 273 (+1), optical, X-ray, radio (+2)**
  - b. Why is this DSO significant compared to other objects of its type? (1 pt) **It was the first quasar to ever be identified, and is one of the most luminous quasars known**
  - c. Why does this object have broad  $H\alpha$  lines? (1 pt) **They are broad due to the rapid rotation of the accretion disc around the central black hole.**
  - d. At visible wavelengths, how many times more luminous is this DSO than the Sun, assuming that both the Sun and this DSO are at a distance of 10 parsecs? (1 pt) **Over 4 trillion times more luminous**

## Section 2: Multiple Choice/Matching (19 pts)

1. Match each characteristic with its corresponding spectral type (OBAFGKM). Spectral types may be used more than once. (5 pts) (0.5 pts per correct answer)

- a. Comprises about 3% of main sequence stars in the solar neighborhood. F
- b. Particularly of interest in the search for extraterrestrial life. K
- c. Rare type of star but comprises 4 of the 90 brightest stars as seen from Earth. O
- d. Luminous and blue, and its spectra has neutral helium lines and moderate hydrogen lines. B
- e. Hydrogen lines are usually absent, but it has titanium oxide bands. M
- f. Surface temperatures of between 10,000 K and 30,000 K. B
- g. Strong hydrogen lines and lines of ionized metals are present. A
- h. Includes our Sun. G
- i. Comprises about 1/13 main sequence stars in the solar neighborhood. G
- j. Between 1.4 and 2.1 solar masses. A

2. What percentage of the mass-energy of the universe is made up of dark energy? (1 pt)

- a. 23%
- b. 68%
- c. 96%
- d. 42%

3. Quantum field theory predicts the value of the cosmological constant ( $\Lambda$ ) to be \_\_\_\_\_ orders of magnitude larger than we observe. (1 pt)

- a. 5
- b. 20
- c. 60
- d. Over 100

4. Which of the following is NOT a way that galaxies are organized? (1 pt)

- a. Clusters
- b. Groups
- c. Filaments
- d. All of these are ways galaxies are organized

5. What is the name of the upper limit to a neutron star's mass? (1 pt)

## Answer Key

- a. Tolman-Oppenheimer-Volkoff limit
- b. Chandrasekhar limit
- c. Schwarzschild limit
- d. Einstein-Rosen limit

6. What is the most widely accepted explanation for the rapid rotation rate of millisecond pulsars? (1 pt)

- a. The stars they formed from rotated, and due to the conservation of angular momentum, its spin rate increased as the mass came closer to the center
- b. Pulsars with an especially strong magnetic field use the vacuum energy of the space around it to increase its rotation speed
- c. They siphon energy from a companion
- d. There is no widely accepted explanation and it is a topic of ongoing research

7. Put the following steps of the cosmological distance ladder in order, from shortest distances to longest. (1 pt) (no partial credit)

Cepheid variables Parallax	Type Ia Supernovae	Gravitationally Lensed Quasars
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- a. \_\_\_\_\_ Parallax \_\_\_\_\_ (shortest)
- b. \_\_\_\_\_ Cepheid variables \_\_\_\_\_
- c. \_\_\_\_\_ Type Ia Supernovae \_\_\_\_\_
- d. \_\_\_\_\_ Gravitationally Lensed Quasars \_\_\_\_\_ (longest)

8. What would an outside observer see as an object approaches and crosses the event horizon of a black hole? (1 pt)

- a. The object would suddenly disappear as it crosses the event horizon
- b. The object would speed up as it approaches the event horizon, before disappearing from view as it crosses
- c. The object would seem to move backwards in time, moving away from the event horizon
- d. The object would appear to move slower and slower as it approaches the event horizon, eventually appearing to freeze right before it crosses

9. Which relationship relates the luminosity of a galaxy to its rotational speed? (1 pt)

- a. Tully-Fisher relationship
- b. Kormendy relationship
- c. Faber-Jackson relationship

## Answer Key

- d. CM relationship
10. What is the average temperature of the Cosmic Microwave Background? (1 pt)
- a. 27.25 K
  - b. 2.725 K
  - c. 13.55 K
  - d. 1.355 K
11. About what percent of the universe's baryons are believed to exist in the Warm-Hot Intergalactic Medium? (1 pt)
- a. 20%
  - b. 5%
  - c. 70%
  - d. 40%
12. Population I stars are \_\_\_\_\_ and \_\_\_\_\_, while Population II stars are \_\_\_\_\_ and \_\_\_\_\_. (1pt)
- a. Older, less luminous; younger, more luminous
  - b. Older, more luminous; younger, less luminous
  - c. Younger, more luminous; older, less luminous
  - d. Younger, less luminous; older, more luminous
13. Which stellar population is more common in globular clusters? (1 pt)
- a. Population I
  - b. Population II
  - c. Population III
  - d. They are all about equally common in globular clusters
14. The motion of the Andromeda galaxy towards the Milky Way galaxy despite the overall expansion of space is an example of which of the following? (1 pt)
- a. Proper motion
  - b. Peculiar motion
  - c. Tangential motion
  - d. Angular motion
15. Which of the following characterizes a starburst galaxy? (1 pt)
- a. Frequent supernova explosions
  - b. Relativistic jet originating at the core of the galaxy
  - c. An extremely high rate of star formation

- d. A low number of protostars

### Section 3: Short/Long Answer (38 pts)

1.
  - a. The Rayleigh-Jeans Law is an approximation of the spectral radiance of electromagnetic radiation based on the wavelength received from a blackbody at a given temperature. This law works well at large wavelengths, however it fails at shorter wavelengths. What is the name of this discrepancy between the observations of shorter wavelengths and the predictions of classical physics? (1 pt) **ultraviolet catastrophe**
  - b. The resolution of this issue came in 1900 with the derivation of a new law that gave the correct radiation at all frequencies. What is the name of this law? (1 pt) **Planck's Law**
2. What is the fusion process that occurs in lower-mass stars that fuses hydrogen into helium? What is the fusion process that occurs in higher-mass stars that fuses heavier elements? (2 pts)  
**Proton - proton chain (+1); CNO cycle (+1)**
3. The upper limit of a neutron star's mass is about 2.16 solar masses. Above this limit, it is thought that neutron degeneracy pressure will fail and the neutron star will collapse into a black hole, however the smallest observed mass of a black hole is around 5 solar masses. What kinds of stellar remnants are theorized to exist with masses between 2.16 and 5 solar masses? (2 pts)  
**Quark stars (+1) and electroweak stars (+1)**
4.
  - a. The No Hair theorem states that a black hole can be completely characterized by three externally observable characteristics. What are these three characteristics? (1 pt) **mass, charge, spin**
  - b. In general relativity, the Schwarzschild solution to Einstein's field equations assumes two of the three external characteristics of a black hole are 0. Which two characteristics are they? (1 pt) **charge and spin**
  - c. In the physical world, the Schwarzschild solution itself as a Schwarzschild black hole. Why is it unlikely that a Schwarzschild black hole actually exists in the universe? (2 pts)

## Answer Key

Even making the assumption that the black hole happened to have no angular momentum when it collapsed, any matter that falls into it will transfer its angular momentum to the black hole, leading to some amount of spin. (+2)

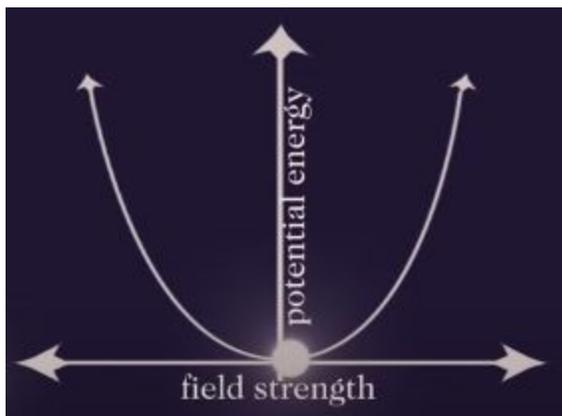
5. The theory of inflation was driven by three major mysteries in cosmology, and inflation solves these mysteries very nicely. These three mysteries are described below:

- Opposite ends of the observable universe have nearly the same temperature and density, implying that they were once close together and were able to freely mix, however, the universe appears too young to have allowed that to happen.
- The curvature of spacetime is almost perfectly flat, which is extremely unlikely to occur by random chance.
- Astronomers believe the early universe was almost perfectly isotropic and homogeneous, however, lumps of matter must have existed to form today's stars and galaxies.

Inflation solves these mysteries by introducing a brief period of rapid, exponential expansion in the early universe.

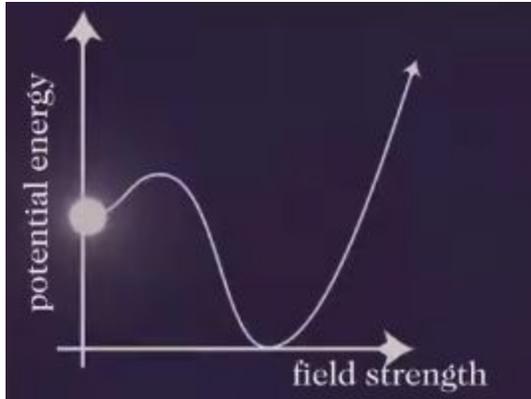
- a. Explain the mechanism that is believed to have caused inflation. (5 pts)

In quantum field theory, space is filled with quantum fields where every point is assigned a value. The "vacuum state" of a field is where the field has the lowest possible energy and is generally correlated with the field strength, as seen in the following graph (+1).



The field theorized to have caused inflation, the inflaton field (may actually be the Higgs field, however this is a subject of debate among physicists), is theorized to have a different correlation between the strength of the field and the energy of the field, a correlation that causes a positive potential energy value, when the field strength is 0 (+1). This correlation can be seen in the following graph.

## Answer Key



The effect of the field strength being at 0 but still having a positive energy value is called a “false vacuum” (+1) and creates a negative pressure that causes superluminal, exponential expansion. Random quantum fluctuations can then cause the energy of the vacuum to truly fall to 0, ending exponential expansion (+1). It is theorized that small parts of the universe fall off from this expansion and expand at a non-exponential rate, while the rest continues to expand exponentially (+1). This is the idea of eternal inflation, and suggests that there are many other universes, possibly with different laws of physics, and that inflation is still occurring outside of our “bubble” universe.

The following link gives a more in-depth explanation:

<https://www.youtube.com/watch?v=xJCX2NlhTc>

b. Explain how inflation solves each of the three mysteries. (6 pts)

1. The exponential expansion of inflation means that opposite ends of the universe were once located right next to each other, allowing for them to mix, which causes their temperatures and densities to be approximately equal (+1). Inflation then carried these sections of the universe away from each other at speeds greater than  $c$  (+1).
2. Any curvature the universe may have had would be essentially “flattened” by inflation (+2). This can be thought of as blowing up a beach ball to the size of the Earth. When it is the size of a beach ball, the curvature is very obvious, however, localized areas (such as the observable universe) seem nearly flat when it is blown up to the size of the Earth.
3. Quantum fluctuations causing minor density changes at the exact moment of inflation would be blown up to much large scales, causing lumps in the universe that would eventually evolve into today’s stars and galaxies (+2).

6. Explain both models of how Type Ia supernovae occur. (2 pts)

1. A white dwarf accretes matter from a companion until it reaches the Chandrasekhar limit and electron degeneracy pressure is overcome, causing it to explode in a Type Ia supernova. (+1)

## Answer Key

2. Two white dwarfs merge, causing electron degeneracy pressure to be overcome and a Type Ia supernova to occur. (+1)

7. Name the mechanism that prevents a white dwarf from collapsing, and name the principle in quantum mechanics that causes this. (2 pts)

Electron degeneracy pressure (+1); Pauli Exclusion Principle (+1)

8. Explain how gravitationally lensed quasars can be used to calculate Hubble's Constant. (2 pts)

When a quasar is brought into our view through gravitational lensing, there's usually multiple images of the quasar due to the different paths of light that were bent by the gravitational lens. These different paths of light have different distances to travel before reaching us. A quasar also has a changing brightness, so there is a time delay between the different images of the quasar, depending on how far the light had to travel. Astronomers can measure this time delay and use modelling of various light paths to determine the expansion rate of the universe, the Hubble Constant. (+2)

9. Refer to Image J.

- a. What is the name of the points labelled on the image? (1 pt)

Lagrange Points

- b. Which of those points are stable? (1 pt)

L4 and L5

10. From what event in the early universe did the CMB photons originate from? (1 pt)

Photon decoupling

11. What is the heaviest element that can be formed inside of a star? (1 pt)

Iron

12. Explain what Hawking radiation is and the mechanism that causes it. (3 pts)

Hawking radiation is a faint radiation emitted by a black hole due to quantum effects near the event horizon (+1). This phenomenon is frequently thought of as occurring due to virtual particle pairs being separated when forming near the event horizon of the black hole. One particle falls into the black hole while the other is launched in the opposite direction, using mass and energy from the black hole, effectively lowering the mass-energy of the black hole itself. (+2)

13. Explain what the information paradox is and why it is such an issue. (4 pts)

When anything falls into a black hole, it appears that its information is "lost," as a black hole emits Hawking radiation that causes the black hole to evaporate slowly, however the Hawking

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radiation does not carry information regarding the contents of the black hole (+2). This is an issue because quantum mechanics dictates that nothing, including information, can be destroyed or lost, however a black hole seems to violate this (+2).

### Section 4: Calculations (41 points)

\*\*\*full credit to correct answers, with or without work. Partial credit may be awarded to incorrect answers that provide some correct work. No credit to incorrect answers with no work.

1. Refer to image K. A planet orbiting around its star, Star A, has an orbital eccentricity of  $e = 0.67$  and a semimajor axis of  $a = 2.3$  AU, and is in a position so that  $\theta = 49.2^\circ$ .

a. What is the distance between Star A and the planet? (3 pts)

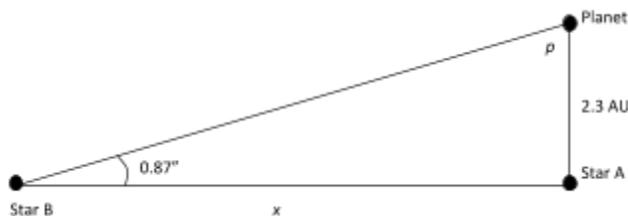
We are given the eccentricity  $e$ , semimajor axis  $a$ , and angle  $\theta$ . We can use the following equation to find the distance  $r$  between Star A and the planet:  $r = \frac{a(1-e^2)}{1+e \cos \theta}$   
After inputting the given variables and solving, we get  $r = 0.882$  AU.

b. What is the orbital period of the planet, in years? (2 pt)

We know the semimajor axis of the orbit, so we can use Kepler's Third Law to find the orbital period of the planet. Since we're dealing with a planet and a star, we can assume  $M_{\text{Star}} \gg M_{\text{Planet}}$ , so we can use the simplified form of Kepler's Third Law:  $P^2 = a^3$   
After substituting 2.3 AU for  $a$ , we find that the orbital period of the planet is 3.49 years.

c. An observer on this planet measures a parallax angle for a distant star, Star B, to be  $0.87''$ . What is the distance between Star A and Star B? (2 pts)

Because this isn't dealing with parallax as observed from Earth, we can't use the familiar parallax equation. Instead, we can solve this using simple trigonometry. Consider the following diagram.



Knowing that there are 3600 arcsec in a degree, we can convert  $0.87''$  to  $2.4167 \times 10^{-4}^\circ$ . Using basic right triangle properties, we can find that angle  $p = 89.99975833^\circ$ . The law of sines can now be used to find length  $x$ . After these calculations, we find that  $x = 545290.24$  AU.

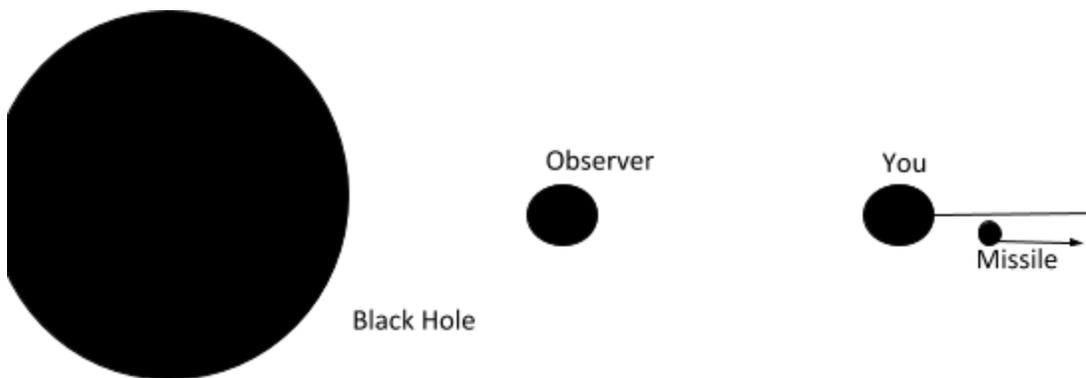
## Answer Key

2. Due to a series of mishaps, you find yourself a mere 140 km away from the center of a 20 solar mass black hole.

a. What is the escape velocity, in km/s? (2 pts)

Using the formula for escape velocity,  $v = \sqrt{\frac{2GM}{r}}$ , we find the escape velocity to be 194759 km/s.

b. If you only had the space technology of today, all hope would be lost as there is no way you could reach that escape velocity. Luckily, you happen to be from the future and you're in a spaceship capable of near-lightspeed travel. You set off away from the black hole at the exact velocity calculated in question 2a. You notice some space debris in your path, so you fire a missile at it to clear it. You measure the speed of the missile to be 100,000 km/s in the positive x direction. An observer located directly behind you sees you and the missile travelling in the positive x direction. This situation can be seen below:



What is the velocity of the missile according to the observer? (6 pts)

We can use the formula for relativistic velocity addition to solve this. The equation is  $u = \frac{v+w}{1+\frac{vw}{c^2}}$

where  $u$  is the speed of the missile as seen from the observer,  $v$  is your velocity, and  $w$  is the speed of the missile as observed by you.

Inputting the values we know (your velocity and the speed of the missile as observed by you), we find that  $u = 242,261$  km/s.

3. You wake up in a different universe where most of the laws of physics are the same, however it hasn't followed the exact same evolution that our universe has. The distance to a galaxy is 32.4 Mpc, and the galaxy is found to have a recessional velocity of 1586.7 km/s.

a. What is the redshift of this galaxy? Either redshift equation (relativistic or approximate) is acceptable. (2 pts) Using the redshift equation accounting for relativistic effects,

$z = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$ , we get  $z = 0.00530675$ . Because 1586.7 km/s is a very small fraction of the

## Answer Key

speed of light, it is acceptable to use the approximation  $z = v/c$  for small velocities. Using this equation, we get  $z = 0.00529267$ . Either answer is acceptable.

- b. If we found a galaxy with the same recessional velocity in our universe, how far away would it be, in Mpc? (2 pts) We can once again use the Hubble law, and using a Hubble Constant of 73.8 km/s/Mpc, we find that the galaxy would be 21.5 Mpc away.
- c. What is the value of the Hubble Constant in this universe? (2 pts) Using the Hubble Law,  $v = Hd$ , we find that  $H = 48.972$  km/s/Mpc.

- d. How old is this universe? (2 pts) The following equation can be used to find the age of

$$t_0(\text{GY}) = \frac{976.01395}{H_0}$$

the universe based on the Hubble Constant:

Applying the answer from 3b, we find that the age of this universe is 19.93 billion years.

4. A star located 17.6 pc away is observed to have an apparent magnitude of 6.1.

- a. What is the absolute magnitude of this star? (2 pts) Using the distance equation,  $m - M = 5 \log\left(\frac{r}{10}\right)$ , we can find that the absolute magnitude is 4.87.

- b. If this star is on the main sequence, which two spectral classes might it belong to? (1 pt) By looking at an HR diagram, we can find that a star with an absolute magnitude of 4.87 on the main sequence is most likely a class F or G star.

- c. Using an HR diagram, estimate the surface temperature of the star. What is the peak wavelength of this star? A range of answers will be accepted. (2 pts) Looking at an HR diagram, the surface temperature of this star could range from about 5500 K to 6750 K.

$$\lambda_{\max} = \frac{b}{T}$$

Using Wien's Law,  $\lambda_{\max} = \frac{b}{T}$ , we find that the peak wavelength of this star is between 429.30 nm and 526.87 nm.

## Answer Key

- d. Calculate the radiant heat energy in watts emitted per meter<sup>2</sup> for this star. (2 pts) Using the same range of temperatures as in the previous question, we can use the

Stefan-Boltzmann equation,  $F = \sigma T^4$ , and find that  $F$  is between  $5.19 \cdot 10^7$  w/m<sup>2</sup> and  $1.18 \cdot 10^8$  w/m<sup>2</sup>.

5. Consider a white dwarf with a mass of 1.1 solar masses and a companion star with a mass of 0.9 solar masses. They are part of a binary system and have a period of 9 hours and 24 minutes.

- a. What is the semimajor axis of their orbit? (2 pts)

Because we know the period and mass of the bodies, we can use Kepler's Third Law to find the semimajor axis. If we convert units to meters, seconds, and kilograms, we can use the following equation:  $P^2 = a^3 \frac{4\pi}{G(M_1+M_2)}$ . After converting and substituting values in, we find that the semimajor axis is  $2.892 \cdot 10^9$  m.

- b. The white dwarf is accreting matter from its companion at a rate of  $4.25 \cdot 10^{22}$  kg per year. How many years will it take before the white dwarf goes supernova? Assume a constant accretion rate. (3 pts) We know that white dwarfs cannot exceed the Chandrasekhar limit at about 1.4 solar masses. Since the white dwarf here needs 0.3 solar masses before reaching this limit, simply convert 0.3 solar masses into kilograms, and divide by its rate of accretion, giving an answer of 14.04 million years before it goes supernova.

6. A Schwarzschild black hole has a mass of 7.2 solar masses.

- a. Calculate the Schwarzschild radius of the black hole. (2 pts) Using the equation for the

Schwarzschild radius,  $r_s = \frac{2GM}{c^2}$ , we find that the Schwarzschild radius of this black hole is 21.27 km.

- b. Calculate the radius of the innermost stable circular orbit of the black hole. (2 pts) The radius of the ISCO is simply 3 times the Schwarzschild radius, so for this black hole, the radius of the ISCO is 63.81 km.

- c. Calculate the radius of the photon sphere of the black hole. (2 pts) The radius of the photon sphere is 1.5 times the Schwarzschild radius, so the photon sphere radius is 31.905 km.