Hovercraft Answer Sheet

Section 1 (28 points)

1. a. Conservation of Energy (1)
   Conservation of Momentum (1)
   Conservation of Angular Momentum (1)
   (Conservation of Matter/Mass accepted)
   b. Conservation of Energy (1)

2. _0 m__ (2)
3. __F=ma______ (2)
4. __10kg______ (2)
5. (i) Speed/Velocity (1)
   (ii) Surface Area (1)
   Also accepted: Shape, Air Density

6. (i)____Scalar_____ (1)
   (ii)___Vector______ (1)

7. __25 Ns (or kgm/s)_ (2)
8. (i)___10 N________ (1)
   (ii)__Opposite______ (1)

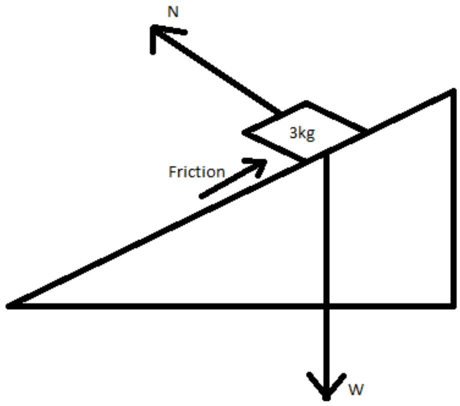
9. __Newton's Third Law__ (2)
10. _They both hit at the same time_ (2)
11. __No__ (2)
12. (i)_Conserved_ (1)
    (ii)_Lost______ (1)
13. (i)__8.85m/s__(1)
    (ii)_78.4 J_____(1)

Section 2 (21 points)

14. i. __**F**__ ii. __**F**__ iii. __**T**__ iv. __**T**__
    v.__**T**__ vi. __**T**_ vii. __**F**__ (1 each)
15. __B___ (2)
16. __A___ (2)
17. __D___ (2)
18. __C___ (2)
19. __C___ (2)
20. __E___ (2)
21. __C___ (2)

Section 3 (43 points)

22. a. Draw diagram below (3)



Normal perpendicular to inclined plane (1)
Force of gravity straight down (1)
Force of friction parallel and going up (1)
(Components of weight, etc are fine)
   b. (i)_1.5 m/s^2_(2)
   (ii)_1.44 s__ (2)
   c. _0.535m__ (6)
   P.C. Law: Conservation of Energy (1)
   Equation: $KE = PE_g + PE_{sp}$ or similar (1)
   (PC ONLY IF PART C IS WRONG)

23. __5.44 x 10^{-4} m (0.000544m)_ (4)
24. __16 m^3/s (NOTE: This should've been 16/pi based on the way I wrote the question. I screwed up and didn't notice until the day after competition, but I checked and this should not have impacted anyone's final placing. Few teams had either of these answers, and those who did were separated from other teams by a significant enough margin.) (3)
25. __M_1sin(theta)__ (4)
26. a. __62.61 m/s_ (3)
    b. __346.423 m_(3)

27. a. __4.42 m_ (3)
    b. _1.44 m/s_ (4)

28. _[$2v_0$/g][1/(1-k)]__ (6)

Partial Credit for Number 28

+2 for [$2v_0$/g] or similar in answer
+4 TOTAL for an answer of:
[$2v_0$/g][1+k+k^2+k^3+… +k^n] or equivalent

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29. Challenge Problem (8 points)

This problem is actually just a complex kinematics problem. Note that there are many ways to solve this problem, but I will present what I consider to be the simplest. As with most two-dimensional kinematics problems, the first step is to set up a table, listing both horizontal and vertical components.

	Horizontal (x)	**Vertical (y)**
Initial Velocity	$V_i\cos(\Theta)$	$V_i\sin(\Theta)$
Final Velocity	$V_i\cos(\Theta)$	
Acceleration	0	-g
Displacement	dcosϕ	dsinϕ
Time		

The tricky part is the displacement. Note that the projectile must land on the inclined plane. This position on the plane can be given by the coordinates (dcosϕ, dsinϕ). The displacement of the projectile will be dcosϕ units in the x-direction and dsinϕ units in the y direction by breaking up the inclined plane into components.

From here, the next task is to solve for time, since the time is the same in both the horizontal and vertical columns. Using the horizontal data, we have: $dcos(\varphi) = v_i \cos(\theta)\, t$. Solving for time yields $t = \frac{dcos(\varphi)}{v_i \cos(\theta)}$. From here, we move to the vertical information. From the vertical data,

$dsin\varphi = v_i \sin(\theta)\, t - \frac{1}{2} g t^2$. Substituting in time, we have $dsin(\varphi) = \frac{v_i \sin(\theta) dcos(\varphi)}{v_i \cos(\theta)} - \frac{gd^2 cos^2(\varphi)}{2{v_i}^2 cos^2(\theta)}$. Notice that we can cancel one *d* in each term of the equation, and also the v_i in the first fraction on the right-hand side. After cancellation, we have $\sin(\varphi) = \frac{\sin(\theta)\cos(\varphi)}{\cos(\theta)} - \frac{gdcos^2(\varphi)}{2{v_i}^2 cos^2(\theta)}$. Next, we know that we want to solve for *d*, so we will move the term with *d* to one side and all other terms to the other side. This results in $\frac{gdcos^2(\varphi)}{2{v_i}^2 cos^2(\theta)} = \frac{\sin(\theta)\cos(\varphi)}{\cos(\theta)} - \sin(\varphi)$. Now we want to get *d* by itself. Algebra yields $d = \frac{2{v_i}^2 cos^2(\theta)\sin(\theta)\cos(\varphi)}{\mathbf{g}cos^2(\varphi)\mathbf{cos}(\theta)} - \frac{2{v_i}^2 cos^2(\theta)\sin(\varphi)}{gcos^2(\varphi)}$. We finally have *d* by itself! Now we have to simplify. Note that we can cancel a cos(Θ) in the first term on the right-hand side, so we will go ahead and do that. Now we get $d = \frac{2{v_i}^2 cos(\theta)\sin(\theta)\cos(\varphi)}{gcos^2(\varphi)} - \frac{2{v_i}^2 cos^2(\theta)\sin(\varphi)}{gcos^2(\varphi)}$. These two terms can be subtracted since they have common denominators, so we get: $d = \frac{2{v_i}^2 cos(\theta)\sin(\theta)\cos(\varphi) - 2{v_i}^2 cos^2(\theta)\sin(\varphi)}{gcos^2(\varphi)}$. Now we factor out a $2{v_i}^2 cos(\theta)$ from the numerator to get $d = \frac{2{v_i}^2 cos(\theta)\,[\sin(\theta)\cos(\varphi) - cos(\theta)\sin(\varphi)]}{gcos^2(\varphi)}$. This equation allows us to use the trigonometric identity that I provided you on the test. $\sin(\theta)\cos(\varphi) - cos(\theta)\sin(\varphi) = \sin(\theta - \varphi)$. So, substituting this into the numerator gives us our final solution:

$$d = \frac{2{v_i}^2 cos(\theta)\sin(\theta - \varphi)}{gcos^2(\varphi)}$$

It is interesting to note that $\lim_{\varphi \to 0} \frac{2{v_i}^2 cos(\theta)\sin(\theta - \varphi)}{gcos^2(\varphi)}$ is just equal to $\frac{2{v_i}^2 sin(2\theta)}{g}$, which is the range equation for when the ground is flat. This is reassuring, since we would expect that as the inclined plane decreases in incline, the equation would approach that of the normal range equation for flat ground.