Exploring the World of Science

Inaugural University of Michigan Science Olympiad Invitational Tournament

Hovercraft

Test length: 50 Minutes

Team number: _____

Team name: _____

Student names: _____

Instructions: Do not open this test until told to do so. You will have 50 minutes to complete this test, but you will be called up at some point during the hour to test your device. You may write on this test, but only answers written on the answer sheet will be graded. The test is broken up into three sections. Section 1 is fill-in-the-blank and quick calculations. Section 2 is true/false and multiple choice. Section 3 is problem-solving. At the end, there is a challenge problem worth 8 points. Feel free to take apart the test, and don't forget units! Good luck, and feel free to ask me any questions you may have!

Useful Equations and Constants:

$g = 9.8\ m/s^2$ on Earth unless otherwise stated

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $ax^2 + bx + c = 0$

Series Convergence: $1+x+x^2+x^3+x^4+\ldots = 1/(1-x)$ for $0 \leq x < 1$

Section 1: 28 pts **Section 2:** 21 pts **Section 3:** 43 pts **Challenge Problem:** 8 pts **Total:** 100 pts

Section 1: Fill-in-the-Blank and Quick Calculations (28 points)
Use the blanks provided on the Answer Sheet to answer the following questions.

1. a. List three major conservation laws of classical mechanics. *(3 points)*
   b. Bernoulli's Principle is really just a statement of one of these laws. Which one? *(1 point)*

2. A ball starting on the ground is launched straight up to a height of 4 m, and then falls straight down back to the ground. What is the ball's displacement? *(2 points)*

3. Write the equation commonly associated with Newton's Second Law. *(2 points)*

4. The mass of a bowling ball on Earth is 10 kg. The acceleration due to gravity on Planet X is 4.9 m/s^2. What is the mass of the bowling ball on Planet X? *(2 points)*

5. List two properties that influence the drag force on an object falling through air. *(2 points)*

6. The mathematical difference between distance and displacement is that distance is a ___**(i)**_____ quantity while displacement is a ___**(ii)**_____ quantity. *(2 points)*

7. A baseball bat applies a force of 500 N to a 0.25kg baseball for 0.05 seconds. What is the impulse experienced by the baseball? *(2 points)*

8. Suppose I'm riding a skateboard and I push off against the ground with a 10 N force. Then, the ground applies a force of ___**(i)**____ on me in the _____**(ii)**_____ direction of my initial force. *(2 points)*

9. Question 8 is an example of which law? *(2 points)*

10. Standing on top of a very tall building, I drop both a bowling ball and a piece of printer paper from the same height at the same time. In the absence of air resistance, which will hit the ground first? *(2 points)*

11. An object is undergoing uniform circular motion. Does it have a constant velocity? *(2 points)*

12. In a perfectly elastic collision, kinetic energy is ____**i: gained/conserved/lost**____. In an inelastic collision, kinetic energy is ___**ii: gained/conserved/lost**____. *(2 points)*

13. An object of mass m=2.0kg is dropped from rest at a height of 4.0m above the ground.
    (i) What is its velocity when just before it hits the ground? *(1 point)*
    (ii) What is its kinetic energy just before it hits the ground? *(1 point)*

Section 2: Multiple Choice and True/False (21 points)
Select the best answer to each question. True/False questions are 1 point each, Multiple Choice are 2 points each.

14. Indicate whether each of the following statements is true (T) or false (F).

    i. If an object is experiencing a net force of zero, its velocity must be zero.

    ii. If an object is experiencing a non-zero net force, its velocity cannot be zero.

    iii. 1 Pascal is equivalent to 1 $kg*m^{-1}s^{-2}$.

    iv. Newton's first law is also known as the law of inertia.

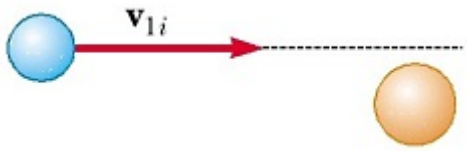
    v. In Fluid Mechanics, liquids, gases, and plasmas are all considered fluids.

    vi. Force is equal to the change in momentum with respect to time.

    vii. The acceleration of an object as produced by a net force is inversely proportional to the magnitude of the net force, in the same direction as the net force, and directly proportional to the mass of the object.

15. The property of a fluid that describes its internal resistance is known as its:
    (A) Friction
    (B) Viscosity
    (C) Resistance
    (D) Internal energy
    (E) Buoyancy

16. An object moving on a frictionless horizontal surface makes a glancing collision with another object initially at rest on the surface as shown below. Which of the following is true about momentum and kinetic energy in the system?



    (A) Momentum is always conserved, and kinetic energy may be conserved.
    (B) Kinetic energy is always conserved, and momentum may be conserved.
    (C) Momentum is always conserved, and kinetic energy is never conserved.
    (D) Both momentum and kinetic energy are always conserved.
    (E) Neither momentum nor kinetic energy is conserved.

17. An object initially at rest accelerates at a rate of 2 m/s^2 for 10 seconds, travelling in a straight line. How far has the object travelled in the 10 seconds?
    (A) 10 m
    (B) 20 m
    (C) 50 m
    (D) 100 m
    (E) 250 m

18. A single particle at rest decays into three smaller particles, all of equal mass, in the absence of external forces. Two particles emerge at a right angle to each other, both with speeds v. What is the speed of the third particle?
    (A) v
    (B) 2v
    (C) $\sqrt{2}v$
    (D) $2\sqrt{2}v$
    (E) The third particle could have many different possible speeds.

19. A ball of mass m falls vertically, hits the floor with speed v_i and rebounds with a speed of v_f. What is the magnitude of the impulse exerted on the ball by the floor?
    (A) $2m(v_f - v_i)$
    (B) $m(v_f - v_i)$
    (C) $m(v_f + v_i)$
    (D) mv_i
    (E) mv_f

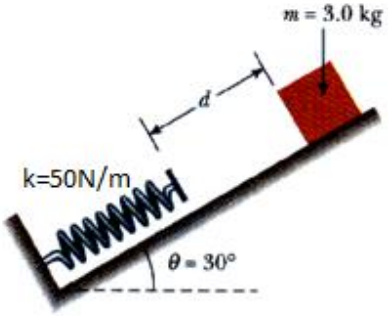
20. A block of mass m_1 is on top of a block of mass m_2. The lower block (the one with mass m_2) is on a horizontal surface with a rope attached that can pull horizontally on it. The coefficient of kinetic friction for all surfaces is μ. What is the resulting acceleration of the lower block if a force F is applied to the rope? Assume that F is large enough that the top block slips on the lower block.
    (A) $a_2 = (F - \mu g(m_2 - m_1))/m_2$
    (B) $a_2 = (F + \mu g(m_1 + m_2))/m_2$
    (C) $a_2 = (F - \mu g(m_1 + 2m_2))/m_2$
    (D) $a_2 = (F - \mu g(m_1 + m_2))/m_2$
    (E) $a_2 = (F - \mu g(2m_1 + m_2))/m_2$

21. Consider a particle at rest which can decay into either two or three smaller particles. Which of the following is true after decaying into two smaller particles, but false after decaying into three smaller particles?
    (A) The total momentum of the smaller particles is zero.
    (B) Given the speed(s) of all but one smaller particle, it is possible to determine the speed of the remaining particle.
    (C) Given the total kinetic energy of the system and the mass of each smaller particle, it is possible to determine the speed of each smaller particle.
    (D) The velocity vectors of the smaller particles must lie in a single plane.
    (E) None of the above

Section 3: Problem Solving (43 points)

All questions can be solved using algebra – no calculus is required. Include appropriate metric units. Don't worry about significant figures. Images are not to scale. Write all of your answers on the Answer Sheet!

22. A block of mass m=3.0kg is moving down the ramp at a velocity of 1.0 m/s at time t=0, and at t=0 it is separated from a spring with spring constant k=50 N/m by a distance *d*=3.0m. The coefficient of kinetic friction between the block and the inclined plane is 0.40 over the distance *d*.

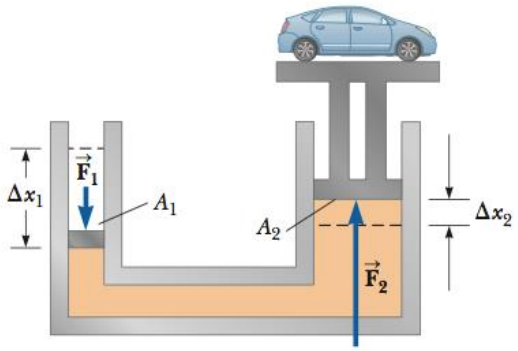


a. Draw a free-body diagram of the forces initially acting on the block (on answer sheet). *(3 points)*

b. (i) What is the acceleration of the block? *(2 points)*
(ii) How long does it take the block to reach the spring? *(2 points)*

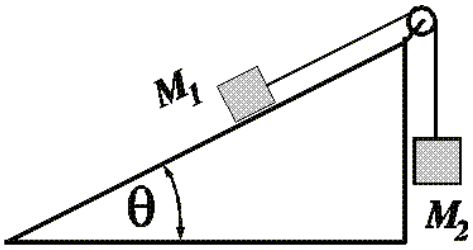
c. How much further will the mass slide until it comes to rest (at least momentarily)? Assume that friction between the block and the inclined plane is negligible once the block hits the spring. *(6 points, tiebreaker if necessary)* Hint: $E_{sp}=0.5kx^2$ where x is the spring's displacement from its starting (resting) point. For two points of partial credit (in the event you don't get the right answer), state the law that you need to use to solve the problem and write an equation based on it in the context of this problem.

23. A hydraulic car lift has a reservoir of fluid connected to two cylindrical fluid filled pipes. The pipe directly below the car has a diameter of 1.5 m. The pipe on which the plunger acts has a diameter of 0.035m. The plunger is depressed a distance of 1.0m. How much does the car rise? *(4 points)*



24. Water is flowing through a tube with diameter 2m at a rate of 4 m^3/s. The tube narrows to a diameter of 1m. What is the velocity of the water in this section? Assume that this fluid is ideal. *(3 points)*

25. Consider the situation depicted in the image below. Both masses are at rest. Neglect all friction.



Given that the ropes and pulley have negligible mass, calculate the mass M_2 in terms of M_1 and the angle theta such that both masses remain at rest. *(4 points)*

26. Consider a cannonball launched from the ground at an angle of 30^o above the horizontal. The maximum height that it reaches is 50m above ground. Neglect air resistance. Assume the ground is perfectly flat everywhere.

    a. What is the initial speed of the cannonball immediately after the launch? *(3 points)*

    b. How far has the cannonball travelled horizontally when it reaches the ground? *(3 points)*

27. A ball of silly putty of mass 0.50 kg is launched from the ground with an initial speed of 10 m/s at an angle of 30^o with the horizontal. At the top of its trajectory, the silly putty strikes a small block of mass 2.5 kg suspended from a string. The silly putty sticks to the block, which can then swing freely. Ignore air resistance, friction, etc.

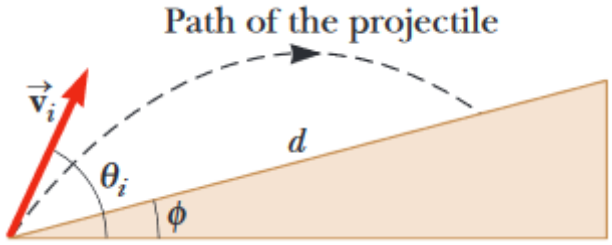
    a. How far has the silly putty traveled horizontally when it reaches the top of its trajectory? *(3 points)*

    b. What is the speed of the system immediately after the collision? *(4 points)*

28. A ball is launched upward from the ground at an initial vertical speed of v_0 and begins bouncing vertically. Every time it rebounds, it loses a proportion of the magnitude of its velocity such that if the speed just before hitting the ground on a bounce is v, then the speed just after the bounce is kv, where $0 < k < 1$ is a constant. Calculate the total length of time that the ball remains bouncing in terms of v_0, g, k, and constants. Assume that any time associated with the contact of the ball with the ground is negligible. Your final answer should be simplified such that the term k only appears once. *(6 points, tiebreaker if necessary)*
    Hint: the series convergence equation will be very useful!

Challenge problem! This problem is harder than the rest! Do your scratch work on this page if you would like, but write your complete proof, with all work you consider important, on the back of the answer sheet. Partial credit will be awarded for correct steps. Nothing on this page will be graded. This problem is worth 8 points and is a tiebreaker.

29. A projectile is launched up an incline (incline angle ϕ) with an initial speed v_i at an angle Θ_i with respect to the horizontal (see the image below) with $\Theta_i > \phi$. Show that the projectile travels a distance $d = \frac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g\cos^2\phi}$ up the incline. *(8 points)*



Calculus is not at all required and will not help. You may find this trigonometric identity to be helpful: $\sin\alpha\cos\beta - \cos\alpha\sin\beta = \sin(\alpha - \beta)$