OPTICS - SCIENCE OLYMPIAD DIVISION C

2017 INVITATIONAL

SCIENCE OLYMPIAD

Exploring the World of Science

You have 50 minutes to complete the test and the laser shoot

SCHOOL/TEAM NAME: ________________________________

TEAM NUMBER: ___________________________________

PARTICIPANT NAMES: ________________________________

Question 1 ___ / 8
Question 2 ___ / 6
Question 3 ___ / 10
Question 4 ___ / 10
Question 5 ___ / 10

Total ___ / 44
1. Good Old Ray Tracing with Lenses

Complete the following ray trace problems. **ALSO: write next to the drawing if the image is: real, virtual, or no image** (2 points each)

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Note: to get full points, they need at least 2 rays for each drawing

1 point for drawing
1 point for writing: "real, virtual, or image"

Real

Real

No image

Virtual
```
2. Good Old Ray Tracing with Mirrors

Complete the following ray trace problems. **ALSO:** write next to the drawing if the image is: real, virtual, or no image (2 points each)
3. Telescopes

(a) Draw a diagram of a refractor telescope. For full points include the following in your diagram: the objective lens, the eyepiece lens, a ray trace for four parallel light rays entering straight into the telescope. (4 points)

(b) If the focal length of the objective lens is 10 meters and the focal length of the eyepiece is 0.25 meters, what is the magnification of the telescope? (2 points)

\[ M = \frac{f_{\text{obj}}}{f_{\text{eye}}} = \frac{10}{0.25} = 40 \]

2 points all or nothing

(c) What kind of aberration does a refractor telescope suffer from? What affect does this aberration have when you look through the telescope? (2 points)

Chromatic Aberration: Index of refraction is slightly different for different wavelengths of light, so different colors are focused differently, creating colored fringes around images.

1 point for identifying chromatic aberration, 1 point for description

(d) Reflector telescopes that use spherical mirrors suffer from another type of aberration. What is this aberration called? Define it. (2 points)

Spherical aberration: A spherical mirror does not perfectly focus incoming light to a single point.

1 point for identifying spherical aberration, 1 point for description.
For this problem, be generous in awarding partial credit. If their answer is equivalent but written differently, that is okay.

4. Fermat’s Principle and the Law of Reflection

Fermat’s Principle says that light will always take the path with the least travel time.

In this problem, you will be deriving the Law of Reflection from Fermat’s Principle.

![Diagram](image)

Your goal is to show that for a light ray shining from point A, bouncing off the surface and traveling to point B, the angles $\theta_i = \theta_r$.

(a) First, come up with an expression of the total length of the reflected path from A to B. (5 points tie breaker)

\[ L = \sqrt{a^2 + x^2} + \sqrt{b^2 + (d-x)^2} \]

(b) Since our speed of light is constant, the path with the least travel time will be the path with the minimum total length. Use a derivative on your answer from part A to minimize total length and use that fact to come up with an equation. From that equation, show that $\sin \theta_i = \sin \theta_r$. From this fact, it follows that $\theta_i = \theta_r$. (5 points)

\[
\frac{dL}{dx} = \frac{1}{2} \frac{2x}{\sqrt{a^2+x^2}} + \frac{1}{2} \frac{2(d-x)(-1)}{\sqrt{b^2+(d-x)^2}} = 0
\]

\[
\frac{x}{\sqrt{a^2+x^2}} = \frac{d-x}{\sqrt{b^2+(d-x)^2}}
\]

\[
\sin \theta_i = \sin \theta_r
\]

\[
\theta_i = \theta_r
\]
5. Fermat’s Principle and Snell’s Law

As in question 3, Fermat’s Principle says that light will always take the path with the least travel time.

Keep in mind that the speed of light is \( v = c/n \) where \( c = \text{speed of light in vacuum} \).

In this problem, you will derive Snell’s Law (Law of Refraction) from Fermat’s Principle.

(a) First, come up with an expression for the total time it takes for the light to travel from point A to point B. (Hint: use the fact that distance = speed \( \times \) time) (5 points tie breaker)

\[
\text{total time } t = \frac{\sqrt{a^2 + x^2}}{c/n_1} + \frac{\sqrt{b^2 + (d-x)^2}}{c/n_2}
\]

(b) To minimize time, take the derivative of the expression from part (a) with respect to \( x \), the vertical distance from A to the point at which the light ray enters the other medium. From this equation, prove Snell’s Law. (5 points)

\[
\frac{dt}{dx} = \frac{x}{c/n_1 \sqrt{a^2 + x^2}} - \frac{(d-x)}{c/n_2 \sqrt{b^2 + (d-x)^2}} = 0
\]

\[
0 = \frac{\sin \theta_1}{c/n_1} - \frac{\sin \theta_2}{c/n_2}
\]

\[
n_2 \sin \theta_2 = n_1 \sin \theta_1
\]