

Answer Key

- | | Martians | Saturnians | | |
|---|--|------------|------|----------|
| • | Mean | 54.7 | 55.0 | 1pt each |
| | Median | 63.3 | 54.3 | |
| | Standard Deviation | 40.1 | 40.5 | |
| | 25 th percentile | 23.4 | 27.3 | |
| | 75 th percentile | 75.9 | 69.3 | |
| | Normal Distribution? | no | ye | |
| | Why? <i>_mean does not equal median for population A; mean is close to median for population B</i> | | | 3 pts |
- | | | |
|----------|--|-------|
| • | Area of room <u>22.85 m²</u> | 4 pts |
| (15 pts) | Perimeter of room <u>29.2 m</u> | 3 pts |
| | Area of carpet <u>25.8 m²</u> | 4 pts |
| | Cost of recarpeting <u>\$1310</u> | 4 pts |
- | | | |
|----------|--------------------------|--------|
| • | Level of water of | 5 pts |
| (15 pts) | Cylinder A <u>150 mL</u> | 10 pts |
| | Cylinder B <u>152 mL</u> | |
- | | | |
|----------|--|-------|
| • | Probability of another sister? <u>slightly > 50%;</u> | 5 pts |
| (10 pts) | <i>4 pts for 50%</i> | 5 pts |
| | Probability of aunt/uncle having 3 girls? <u>1/8=12.5%</u> | |
- | | | |
|----------|---|--------|
| • | Minimum area for 21 coins? <u>12096 mm²</u> | 4 pts |
| (10 pts) | <i>Description of arrangement <u>3x7 or 21x1</u></i> | 3 pts |
| | Minimum area for 50 coins? <u>27403 mm²</u> | 5 pts |
| | <i>Description of arrangement <u>interlaced 5/4</u></i> | 3 pts |
| (15 pts) | <i><u>alternating</u></i> | 15 pts |

85 point total

10 pts for correct criteria for stopping, but incorrect calculations

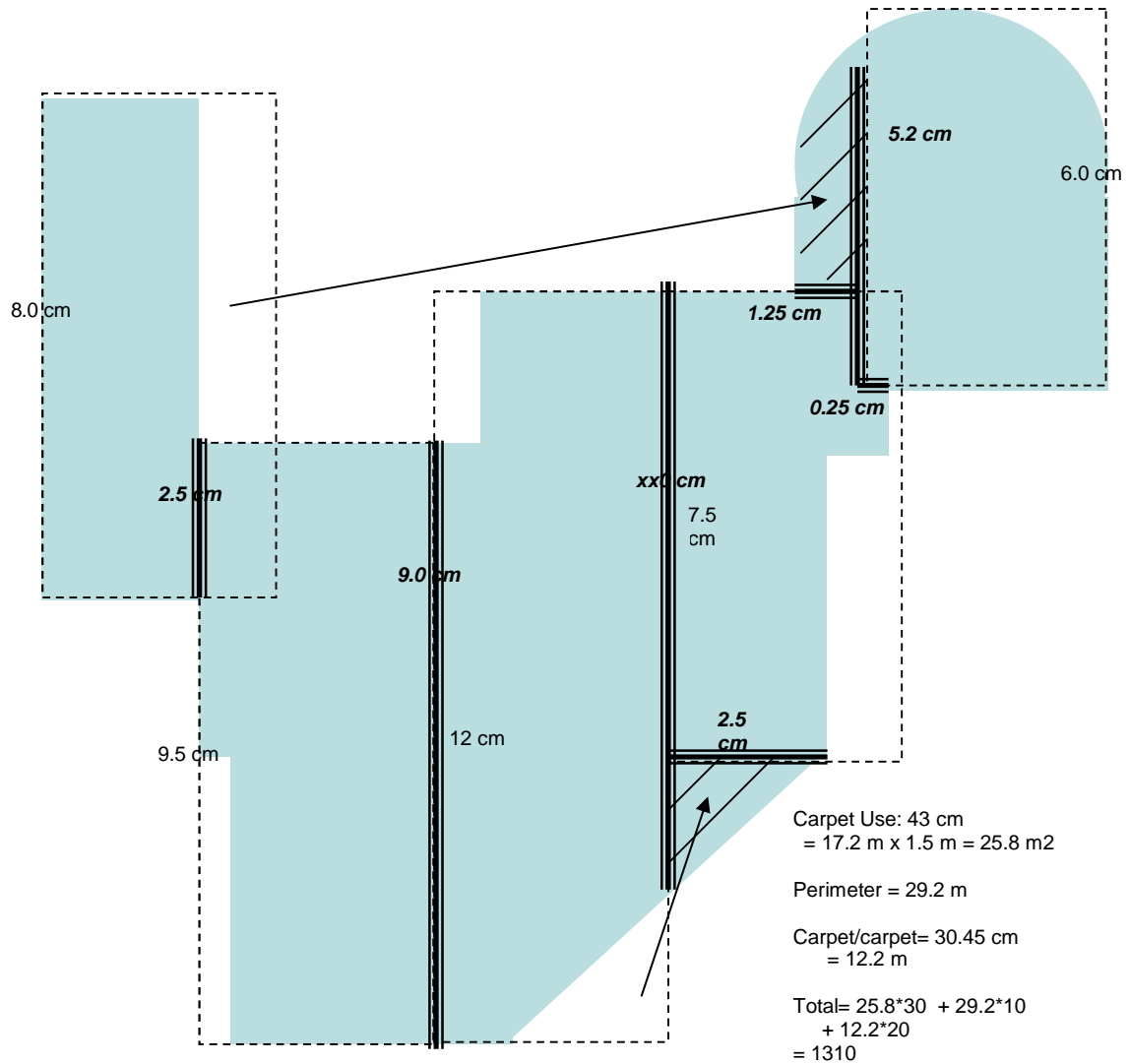
5 points for showing power based approach rather than iterative

1. Find the mean (μ), median, standard deviation (σ) and 25th and 75th percentiles for population A and population B. Do the data seem to be drawn from a normally distributed population? Why or why not?

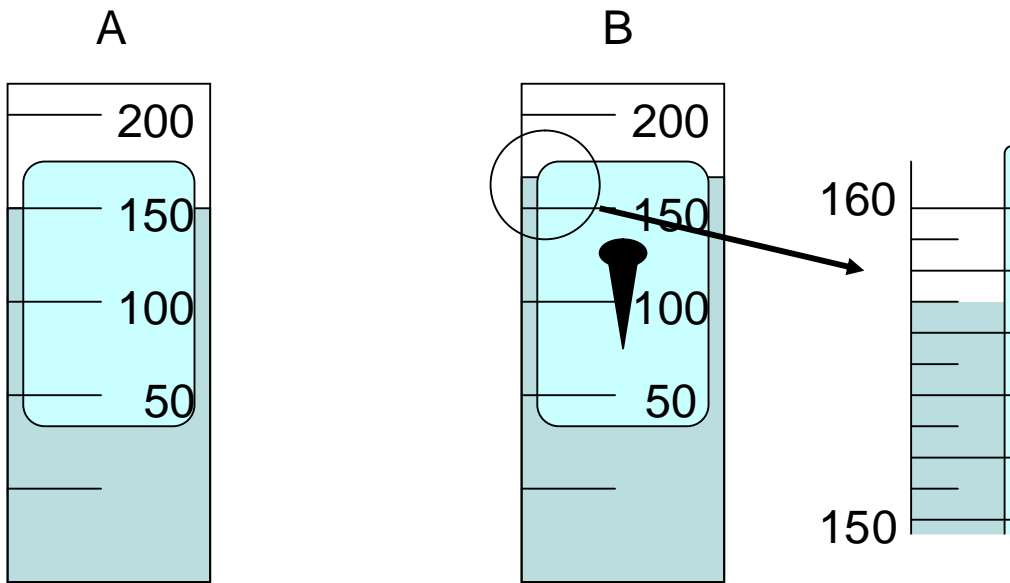
$$\sigma^2 = (\sum(x-\mu)^2)/N$$

	A=Height of Martians	B=Height of Saturnians
	70.5	69.3
	52.2	23.3
	21.9	79.3
	63.3	55.9
	69.1	103.7
	40.1	40.5
	77.3	54.3
	42.9	56.1
	25.6	10.2
	75.9	43.9
	14.0	98.7
	78.3	45.5
	79.3	34.0
mean	54.7	55.0
median	63.3	54.3
25%	40.1	40.5
75%	75.9	69.3
stdev	23.4	27.3

2. You are a homeowner and need to recarpet your part of your house. You are charged based on 3 things. 1) Length of boundary between carpet and wall (\$10/m) 2) length of seam (carpet/carpet) (\$20/m) and 3) total amount of carpet required to cover area (including waste that you do not use) (\$30/m²). Carpet is provided in strips 1.5m wide. The carpet has a directionality, so can only be laid in one orientation. What is the minimum cost to recarpet the space below? Please provide the area of this space as well as the perimeter. Scale= 2.5 cm=1 m. Provide calculations.



3. You are provided with two frozen ice cubes, each contains the same amount of water. In one ice cube, a screw is frozen in the middle. You place the ice cubes into a metric graduated cylinder (containing 100 mL of water) and observe figures A and B when the samples are first put in the water. Knowing that the density of the metal used to make the screw is 3500 kg/m^3 , what will the levels of the water be once the two ice cubes have defrosted? Provide all calculations. Note, only the water levels are drawn to scale.



A: Ice cube is displacing volume equivalent of the weight. When the ice cube melts, the it will displace the same volume. **Final value=150 mL**

Volume of water in ice cube =50 mL

B: When the screw is floating in the ice cube, it is displacing the weight of the screw + water = 157 mL. Water=150 mL; screw=7mL=7g

Once it defrosts, the screw will fall to the bottom of the water and displace its volume.

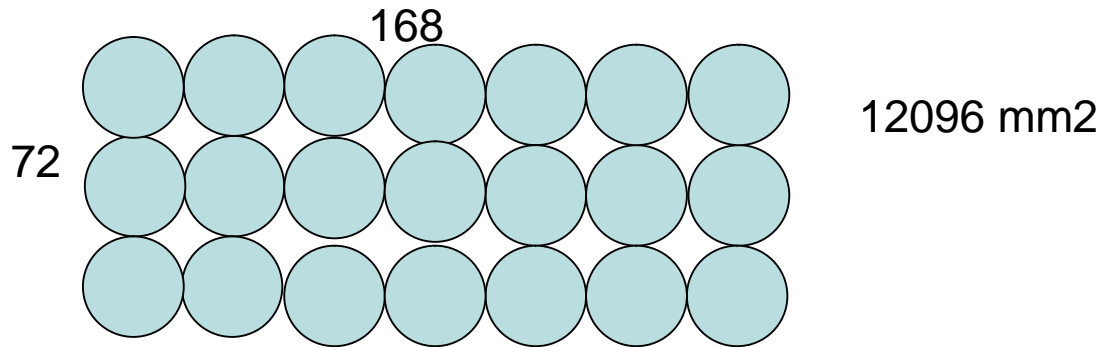
Density=wt/volume \rightarrow volume=wt/density

Density $3500 \text{ kg/m}^3 = 3500 \text{ kg/m}^3 * (1000 \text{ g/kg}) * (1\text{m}^3/10^6 \text{ cm}^3) = 3.5 \text{ g/cm}^3$

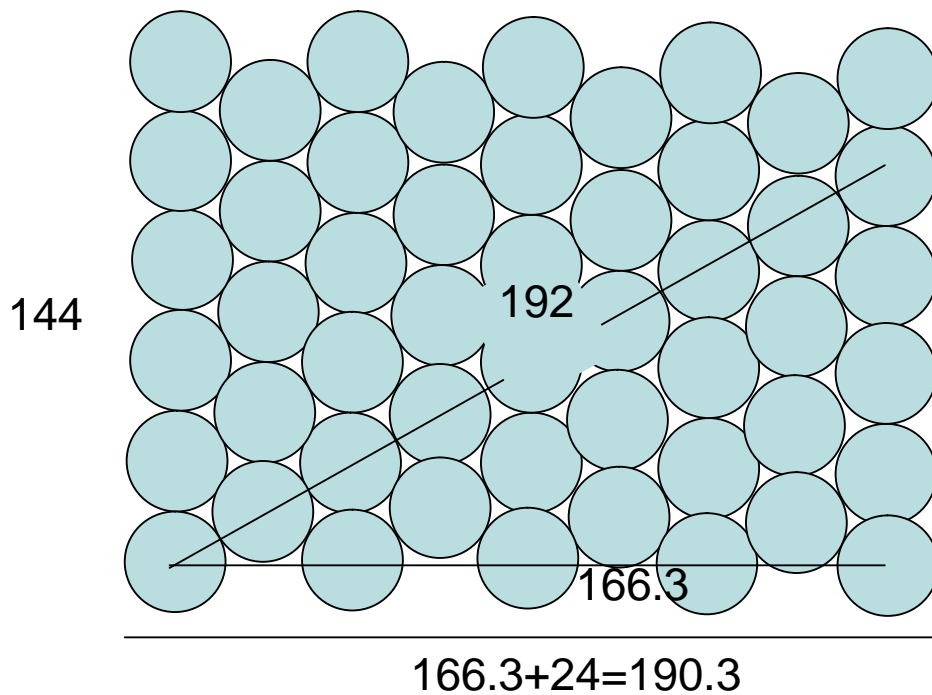
Volume of screw = $7\text{g}/(3.5 \text{ g/cm}^3) = 2 \text{ cm}^3$

Final volume = 152 mL

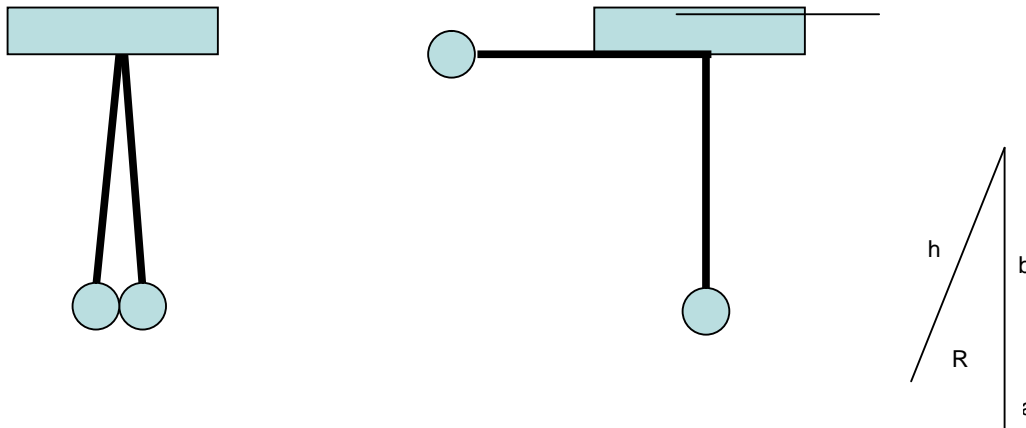
The US mint just released the 21st state quarter into circulation-- the California Quarter!
 Please calculate the smallest area rectangle that you could display all 21 quarters in.
 What is the smallest rectangle you will be able to display them in once all 50 have been
 released in 2008. FYI: the Diameter of a quarter is: 24 mm.



Or one row of 21 12096 mm²



6. Two 3 cm balls are attached to 25 cm strings as shown. One of the balls is raised to the horizontal and released. Assuming that 20% of the energy is lost with each collision, how many times will the balls collide before they stop moving? Note: potential energy = mgh ; kinetic energy = $\frac{1}{2}mv^2$; $g = 9.8 \text{ m/s}^2$



Initial potential energy = mgh
 Collisions stop when potential energy is \leq to that of the balls resting on left
 Final potential energy = mga
 $A = h - \sqrt{h^2 - r^2}$

For each collision 1 $E = 0.8 \cdot mgh$
 collision 2 $E = 0.8 \cdot 0.8 \cdot mgh$
 collision 3 $E = 0.8 \cdot 0.8 \cdot 0.8 \cdot mgh \dots$
 For n th collision, energy $E = 0.8^N \cdot mgh$

Oscillations stop when ...

$$0.8^N \cdot mgh \leq mg(h - \sqrt{h^2 - r^2}) \quad \text{canceling out } m \text{ and } g$$

$$0.8^N \leq \frac{1 - \sqrt{h^2 - r^2}}{h}$$

$$\leq 1 - \sqrt{1 - (r^2/h^2)}$$

Solving for N

$$\log(0.8^N) \leq \log(1 - \sqrt{1 - r^2/h^2})$$

$$N \cdot \log(0.8) \leq \log(1 - \sqrt{1 - r^2/h^2})$$

$$N \leq \frac{\log(1 - \sqrt{1 - r^2/h^2})}{\log(0.8)}$$

Using $r=1.5$, $h=25$ the above equation results in 28 collisions